ALGORITHM FOR NUMERICAL SIMULATION
THE DYNAMICS OF COASTAL BOTTOM RELIEF
USING HIGH-PERFORMANCE COMPUTING

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RELEVANCE
The current ecological state of coastal systems is largely determined by the variety of particulate matter of both mineral and organic origin. Under the influence of a complex of intra-water processes, transformation and sedimentation of particles of suspended matter and, as a result, the formation of bottom sediments occurs.

Sediment of coastal systems is a complex heterogeneous physicochemical system, the study of the processes inside which under modern conditions is an important and urgent task.

In particular, it seems relevant to assess the flows of pollutants and determine the amount of sediment deposited associated with the development of industrial and recreational activities in coastal areas.

As a rule, research in this area requires the construction of mathematical models that are as close as possible to real processes and allow predicting the distribution of suspended matter in an aqueous medium.
To predict changes in the bottom topography, a non-stationary 2D model of sediment transport is used in the work, taking into account the following physically significant factors and parameters: soil porosity; the critical value of the shear stress at which sediment movement begins; turbulent exchange; dynamically changing bottom geometry, wind currents and bottom friction.

A discrete model of sediment transport, proposed as a result of approximation of the corresponding linearized continuous model, is proposed and investigated. The sediment transport model is complemented by a model of the movement of the aquatic environment and turbulence.

Since the mathematical problems corresponding to the constructed models must be solved in real or accelerated time scales, on grids including $10^6$–$10^{12}$ nodes, it is necessary to develop parallel algorithms for computer systems with mass parallelism.

For the proposed models of hydrodynamic processes, parallel algorithms are developed that are implemented as a complex of programs. Numerical experiments are performed for model problems of bottom sediment transport and bottom topography transformation taking into account underwater structures, the results of which are consistent with real physical experiments.
PART I

CONTINUOUS 2D MODEL OF SEDIMENT TRANSPORT
The equation of sediment transport is considered, which can be written in a divergent form:

\[
(1 - \varepsilon) \frac{\partial H}{\partial t} + \text{div} \left( \frac{A\bar{\omega}d}{(\rho_1 - \rho_0)gd} \beta \left| \vec{\tau}_b - \frac{\tau_{bc}}{\sin \varphi_0} \text{grad}H \right|^{\beta-1} \vec{\tau}_b \right) = \\
\text{div} \left( \frac{A\bar{\omega}d}{(\rho_1 - \rho_0)gd} \beta \left| \vec{\tau}_b - \frac{\tau_{bc}}{\sin \varphi_0} \text{grad}H \right|^{\beta-1} \frac{\tau_{bc}}{\sin \varphi_0} \text{grad}H \right),
\]

where \( H = H(x, y, t) \) is the pond depth
\( \varepsilon \) is the porosity of bottom materials
\( \vec{\tau}_b \) is the vector of tangential stress sat the water bottom
\( \tau_{bc} \) is the critical value of the tangential stress at which the transport of bottom materials begins
\( \varphi_0 \) is an angle of repose of soil in the water, \( \tau_{bc} = a\sin \varphi_0 \)
\( \rho_1, \rho_0 \) are density of particles of the bottom material and the aqueous medium, respectively
\( g \) is the gravity acceleration; \( \bar{\omega} \) is the averaged wave frequency
\( A, \beta \) are the dimensionless constants (\( A=19.5; \beta=3 \))
\( d \) are characteristic sizes of soil particles
As with the equation of transport of suspensions, the region of specifying equation (1) is the cylinder $C_Y = D \times (0, T)$, $D(x, y) = \{0 < x < L_x, 0 < y < L_y\}$.

We introduce a coefficient $k = k(x, y, t)$ that depends in a nonlinear way on the partial derivatives with respect to the spatial variables of the function $H = H(x, y, t)$

$$k \equiv \frac{A\omega d}{((\rho_1 - \rho_0)gd)\beta} \left| \vec{\tau}_b - \frac{\tau_{bc}}{\sin \varphi_0} \cdot \text{grad}H \right|^{\beta-1}. \quad (2)$$

In equation (1), the term of the form

$$\left| \vec{\tau}_b - \frac{\tau_{bc}}{\sin \varphi_0} \cdot \text{grad}H \right|$$

is the length of the corresponding vector.

Taking (2) into account, equation (1) can be written in the form:

$$(1 - \varepsilon) \frac{\partial H}{\partial t} + \text{div}(k \cdot \vec{\tau}_b) = \text{div} \left( k \cdot \frac{\tau_{bc}}{\sin \varphi_0} \cdot \text{grad}H \right). \quad (3)$$
NONLINEAR 2D MODEL OF SEDIMENT TRANSPORT

Fig. 1. Estimated area

Fig. 2. Profile of the bottom with a cut along the Oy axis
NONLINEAR 2D MODEL OF SEDIMENT TRANSPORT

Initial conditions of the problem:

\[ H(x, y, 0) = H_0(x, y), H_0(x, y) \in C^2(D) \cap C(\overline{D}) \]
\[ \text{grad}_{(x,y)}H_0 \in C(\overline{D}), (x, y) \in \overline{D} \]  \hspace{1cm} (4)

Conditions on the boundary of the calculated area:

\[ \left. \left| \vec{\tau}_b \right| \right|_{y=0} = 0 \]  \hspace{1cm} (5)
\[ H(0, y, t) = H_1(y, t), 0 \leq y \leq L_y \]  \hspace{1cm} (6)
\[ H(L_x, y, t) = H_2(y, t), 0 \leq y \leq L_y \]  \hspace{1cm} (7)
\[ H(x, 0, t) = H_3(x), 0 \leq x \leq L_x \]  \hspace{1cm} (8)
\[ H(x, L_y, t) = 0, 0 \leq x \leq L_x \]  \hspace{1cm} (9)

In addition to the boundary conditions (5) – (8), we assume that their smoothness conditions are satisfied, the existence of continuous derivatives on the boundary of the region:

\[ \text{grad}_{(x,y)}H \in C(CY_T) \cap C^1(CY_T) \]  \hspace{1cm} (10)

The non-degeneracy condition for the operator of the problem has the form

\[ k \geq k_0 = \text{const} > 0, \quad \forall (x, y) \in \overline{D}, 0 < t \leq T \]  \hspace{1cm} (11)

The vector of tangential stress at the bottom is expressed using unit vectors of the coordinate system in a natural way

\[ \vec{\tau}_b = \vec{i}\tau_{bx} + \vec{j}\tau_{by}, \]  \hspace{1cm} (12)
\[ \tau_{bx} = \tau_{bx}(x, y, t), \tau_{by} = \tau_{by}(x, y, t) \]
The input data for sediment transport model is the vector of the velocity water medium field

The mathematical model of sediment transport is supplemented by models of water movement and turbulence, the equations of which are solved in the hydrodynamic block by the pressure correction method

The spatially inhomogeneous 3D model of hydrodynamics proposed by the author's team is described in the works:


We construct a uniform grid $\omega_\tau$ with the time step $\tau$ (i.e. the set of points $\omega_\tau = \{ t_n = n\tau, n = 1, \ldots, N, N\tau = T \}$) and we realize the linearization of the initial-boundary value problem (3)–(9) to create a linearized model on the time interval $0 \leq t \leq T$.

We linearize the term $\text{div}(k \cdot \vec{v}_b)$ and the coefficient $k$ by choosing their values at the time $t = t_{n-1}, n = 1, \ldots, N$ and considering equation (3) in the time interval $t_{n-1} < t \leq t_n$, $n = 1, 2, \ldots, N$.

It is assumed that we know the function $H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1})$ and its partial derivatives at spatial variables.

In the case, if $n = 1$, it is enough to take the function with the initial conditions $H^{(1)}(x, y, t_0)$, i.e. $H^{(1)}(x, y, t_0) = H_0(x, y)$. If $n = 2, \ldots, N$, the function $H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1})$ is assumed to be known, since it is assumed that the problem (3)–(9) for the previous time interval $t_{n-2} < t \leq t_{n-1}$ is solved.
We introduce the notation:

\[ \mathbf{k}^{(n-1)} \equiv \frac{A\Delta d}{((\rho_1 - \rho_0)gd)}^{\beta} \left| \mathbf{\ddot{r}}_b - \frac{\tau_{bc}}{\sin \varphi_0} \text{grad}H^{(n-1)}(x, y, t_{n-1}) \right|^{\beta^{-1}}. \]  

Then equation (3) after the linearization takes the form:

\[ (1 - \varepsilon) \frac{\partial H^{(n)}}{\partial t} + \text{div}(k^{(n-1)} \mathbf{\ddot{r}}_b) = \text{div} \left( k^{(n-1)} \frac{\tau_{bc}}{\sin \varphi_0} \text{grad}H^{(n)} \right), \quad t_{n-1} < t \leq t_n, \]  

and we supplement it with the initial conditions:

\[ H^{(1)}(x, y, t_0) = H_0(x, y), \quad H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1}), \quad (x, y) \in \overline{D}. \]  

The equation component \( \text{div}(k^{(n-1)} \cdot \mathbf{\ddot{r}}_b) \) is a well-known function of the right side at such linearization. The boundary conditions (5)–(9) are expected to be completed for all time intervals \( t_{n-1} < t \leq t_n, n = 1, 2, \ldots, N \).

Note that the coefficients \( k^{(n-1)}, n = 1, 2, \ldots, N \) depend on the spatial variables \( x, y \) and the time variable \( t_{n-1}, n = 1, 2, \ldots, N \), defined by the choice of step \( \tau \):

\[ k^{(n-1)} = k^{(n-1)}(x, y, t_{n-1}), \quad n = 1, 2, \ldots, N. \]
The existence and uniqueness of the solution of the initial-boundary problem of sediment transport are proved.

An a priori estimate of the solution norm is obtained depending on the integral estimates of the right-hand side, the boundary conditions and the initial condition.


Sufficient conditions for the positiveness of solutions of the linearized sediment transport problem and their convergence to the solution of the original nonlinear norm in the norm of space $L_1$ with the velocity $O(\tau)$ are determined, where $\tau$ is the time step.

Let construct a finite-difference scheme approximating problem (14), (15), (5) - (9). Cover the region $D$ with a uniform rectangular computational grid

$$\omega = \omega_x \times \omega_y$$

$$\omega_x = \{x_i = ih_x, 0 \leq i \leq N_x - 1, l_x = h_x(N_x - 1)\}$$

$$\omega_y = \{y_j = jh_y, 0 \leq j \leq N_y - 1, l_y = h_y(N_y - 1)\}$$

(16)

To obtain a difference scheme, we use the balance method. Integrate both sides of equation (14) over the region $D_{txy}$

$$D_{txy} \in \{t \in [\tau_n, \tau_{n+1}], x \in [x_{i-1/2}, x_{i+1/2}], y \in [y_{j-1/2}, y_{j+1/2}]\},$$

as a result, obtain the following equality:

$$\iiint_{D_{txy}} (1 - \varepsilon) H_t^{(n)} \, dt \, dx \, dy + \iiint_{D_{txy}} (k^{(n-1)} \tau_{b,x})_x \, dt \, dx \, dy + \iiint_{D_{txy}} (k^{(n-1)} \tau_{b,y})_y \, dt \, dx \, dy =$$

$$= \iiint_{D_{txy}} (k^{(n-1)} \frac{\tau_{bc}}{\sin \varphi_0} H_x^{(n)})_x \, dt \, dx \, dy + \iiint_{D_{txy}} (k^{(n-1)} \frac{\tau_{bc}}{\sin \varphi_0} H_y^{(n)})_y \, dt \, dx \, dy.$$ 

(17)
(1 - \varepsilon) \frac{H_{i,j}^{(n+1)} - H_{i,j}^{(n)}}{\tau} + \frac{k_{i+1/2,j}^{(n)}(\tau_{b,x})_{i+1/2,j}^{(n)} - k_{i-1/2,j}^{(n)}(\tau_{b,x})_{i-1/2,j}^{(n)}}{h_x} + \\
\tau \frac{k_{i,j+1/2}^{(n)}(\tau_{b,y})_{i,j+1/2}^{(n)} - k_{i,j-1/2}^{(n)}(\tau_{b,y})_{i,j-1/2}^{(n)}}{h_y} = \\
= \frac{\tau_{bc}}{\sin \varphi_0} \left( k_{i+1/2,j}^{(n)} \frac{H_{i+1,j}^{(n+\sigma)} - H_{i,j}^{(n+\sigma)}}{h_x^2} - k_{i-1/2,j}^{(n)} \frac{H_{i,j}^{(n+\sigma)} - H_{i-1,j}^{(n+\sigma)}}{h_x^2} \right) + \\
+ \frac{\tau_{bc}}{\sin \varphi_0} \left( k_{i,j+1/2}^{(n)} \frac{H_{i,j+1}^{(n+\sigma)} - H_{i,j}^{(n+\sigma)}}{h_y^2} - k_{i,j-1/2}^{(n)} \frac{H_{i,j}^{(n+\sigma)} - H_{i,j-1}^{(n+\sigma)}}{h_y^2} \right)

k_{i+1/2,j}^{(n)} = \frac{A\omega d \left( (\tilde{\tau}_b)_{i+1/2,j}^{(n)} - \frac{\tau_{bc}}{\sin \varphi_0} (\text{grad}H)_{i+1/2,j}^{(n)} \right) \left( \rho_1 - \rho_0 \right) g d}{\left( (\rho_1 - \rho_0) g d \right)^\beta} \int_h \left( (\tilde{\tau}_b)_{i+1/2,j}^{(n)} - \frac{\tau_{bc}}{\sin \varphi_0} (\text{grad}H)_{i+1/2,j}^{(n)} \right) - \tau_{bc}

(\tau_{b,x})_{i+1/2,j}^{(n)} = \frac{(\tau_{b,x})_{i+1,j}^{(n)} + (\tau_{b,x})_{i,j}^{(n)}}{2}

(\tau_{b,y})_{i,j+1/2}^{(n)} = \frac{(\tau_{b,y})_{i,j+1}^{(n)} + (\tau_{b,y})_{i,j}^{(n)}}{2}

(\text{grad}H)_{i+1/2,j}^{(n)} = \frac{H_{i+1,j} - H_{i,j}}{h_x} i + \frac{H_{i+1/2,j+1} - H_{i+1/2,j-1}}{2h_y} j

(\text{grad}H)_{i+1/2,j}^{(n)} = \frac{H_{i+1,j} - H_{i,j}}{h_x} i + \frac{H_{i+1/2,j+1} - H_{i+1/2,j-1}}{2h_y} j
PART II

NUMERICAL EXPERIMENTS FOR MODELING SEDIMENT TRANSPORT AND BOTTOM TOPOGRAPHY DYNAMICS
A software package implemented in C++ is designed to build turbulent flows of an incompressible velocity field of the aquatic environment on high-resolution grids for predicting sediment transport and possible scenarios of changing the geometry of the bottom region of shallow water bodies. Parallel algorithms implemented in the software package for solving systems of grid equations arising during the discretization of model problems were developed using MPI technology.

To solve this problem, we used the adaptive modified alternating-triangular method of minimum corrections. In parallel implementation, decomposition methods of grid domains were used for computationally time-consuming diffusion-convection problems, taking into account the architecture and parameters of a multiprocessor computing system. The decomposition of the calculated two-dimensional region is made in two spatial variables. The peak performance of a multiprocessor computing system is 18.8 teraflops. As computing nodes, 128 HP ProLiant BL685c homogeneous 16-core Blade servers of the same type are used, each of which is equipped with four 4-core AMD Opteron 8356 2.3GHz processors and 32GB RAM.
Figure 1 shows the dependence of acceleration on the number of processors needed to solve the model problem on various grids. The numbering of the graphs corresponds to the following dimensions of the calculation grids: 1 - 100×100, 2 - 200×200, 3 - 500×500, 4 - 1000×1000, 5 - 2000×2000, 6 - 5000×5000.

Fig. 3. Graph of the acceleration of the parallel algorithm (a - based on an explicit scheme, b - based on an implicit scheme)
A graph of acceleration versus the number of processors needed to solve the problems of hydrodynamics is presented in Fig. 4. To calculate the velocity vector of the aqueous medium, computational grids of size $50 \times 250 \times 40$.

**Fig. 4** Schedule of acceleration of the parallel algorithm for solving hydrodynamic problems
A series of numerical experiments was performed to simulate the dynamics of changes in the topography of the bottom of a complex configuration in the coastal zone of a reservoir. In model problems, the presence of obstacles (groynes, underwater breakwaters, breakwaters, dumps, etc.) and various irregularities underlying its surface was assumed to be on the bottom surface.

The simulation section under consideration has dimensions of 55 m × 55 m horizontally and 2 m vertically (in depth), the peak point rises above sea level to 1 m. Suppose that the liquid is at rest at the initial time.

The size of the computational grid is 110 × 110, the step in spatial variables is 0.05 m, the time step is 0.01 s, the wind speed is 5 m/s and is directed from left to right.
Fig. 5. The geometry of the computational domain with structures in the form of groynes
Fig. 6. The geometry of the computational domain with structures in the form of underwater breakwaters
Fig. 7. The geometry of the computational domain with spiky structures in the form of underwater banquets
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